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We present a complete analysis of the fermion masses and mixing matrices in the framework of the little flavon model. In this model textures are generated by coupling the fermions to scalar fields, the little flavons, that are pseudo-Goldstone bosons of the breaking of a global $SU(6)$ symmetry. The Yukawa couplings arise from the vacuum expectation values of the flavon fields, their sizes controlled by a potential à la Coleman-Weinberg. Quark and lepton mass hierarchies and mixing angles are accommodated within the effective approach in a natural manner.

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I. FRAMEWORK

Fermion masses and mixing angles can in principle be explained by patterns of vacuum expectation values of one or more fields (the *flavons*) in spontaneously broken horizontal symmetries [1, 2]. The hierarchies in these expectation values—which must be present in order to explain the experimental data—require however some mechanism in order to be stabilized without resorting to fine tuning of the parameters of the theory.

A possible framework solving this problem of fine tuning of the parameters—and providing an explicit effective potential from which the vacuum expectation values can be derived—has been introduced in ref. [3] where the spontaneous breaking of the horizontal symmetry is driven by the vacuum expectation values (VEV's) of flavon fields originally arising as pseudo-Goldstone bosons of a larger group. The mechanism used in this model to stabilize the breaking scale against one-loop quadratically divergent radiative correction is that of *collective* breaking already employed in the little Higgs models in the context of electroweak symmetry breaking [4].

In the model, the flavons are the pseudo-Goldstone bosons of the breaking of a flavor symmetry group $SU(6)$ down to $Sp(6)$. Fourteen of the generators of $SU(6)$ are broken giving 14 (real) Goldstone bosons that can be written as a single field

$$\Sigma = \exp(i\Pi/f) \Sigma_0. \quad (1)$$

They represent fluctuations around the (anti-symmetric) vacuum expectation value

$$\Sigma_0 \equiv \langle \Sigma \rangle = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}. \quad (2)$$

At the same time, four subgroups $[SU(2) \times U(1)]^2$ defined in ref. [3] are gauged. The gauge transformations explicitly break the global $SU(6)$ symmetry thus giving mass to the pseudo Goldstone bosons. In the low-energy limit, we can write the pseudo-Goldstone boson matrix as

$$\Pi = \begin{pmatrix} 0 & 0 & \phi_1^+ & 0 & s & \phi_2^0 \\ 0 & 0 & \phi_1^0 & -s & 0 & \phi_2^- \\ \phi_1^- & \phi_1^{0*} & 0 & -\phi_2^0 & -\phi_2^- & 0 \\ 0 & -s^* & -\phi_2^{0*} & 0 & 0 & \phi_1^- \\ s^* & 0 & -\phi_2^+ & 0 & 0 & \phi_1^{0*} \\ \phi_2^{0*} & \phi_2^+ & 0 & \phi_1^+ & \phi_1^0 & 0 \end{pmatrix}. \quad (3)$$

The singlet field becomes massive and has no expectation value in the vacuum configuration we will use; it is therefore effectively decoupled from the theory. The two doublets are our *little flavons*:

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad \text{and} \quad \phi_2 = \begin{pmatrix} \phi_2^0 \\ \phi_2^- \end{pmatrix}, \quad (4)$$

that are $SU(2)$ -doublets with $U(1)$ charges respectively $1/2$ and $-1/2$, and one $SU(2)$ - and $U(1)$ - singlet s .

The Coleman-Weinberg effective potential [5], induced by $SU(6)$ non-symmetric gauge and Yukawa interactions has been discussed in [3] and takes the form:

$$\mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_4 |\tilde{\phi}_2^\dagger \phi_1|^2. \quad (5)$$

An explicit analysis of this potential shows that at the leading order the quadratically divergent part contains only the term $\lambda_4 = g_1^2 g_2^2 / (g_1^2 + g_2^2)$ and there are no mass terms for the flavon doublets. This result is in agreement with the general features of the little Higgs models.

The logarithmically divergent part generates the other quartic couplings in eq. (5) with a typical size given by [3]

$$\begin{aligned} \lambda_{1,2} &\simeq \frac{\lambda_4^2}{64\pi^2} \log \frac{\Lambda^2}{M_\phi^2} \lesssim 10^{-2}, \\ \lambda_{1,2,3} &\simeq c_{1,2,3} \frac{3g^4}{64\pi^2} \log \frac{\Lambda^2}{M_V^2} \lesssim 10^{-2}, \end{aligned} \quad (6)$$

where c_i are numerical coefficients, related to the expansion of the Σ , and $M_V \simeq f$ is the mass of the massive gauge bosons. $\Lambda = 4\pi f$ is the model cut-off scale that we take around 100 TeV. Explicit $SU(6)$ breaking in the Yukawa interactions leads to quadratically divergent contributions to flavon mass terms of the form

$$\mu_{1,2}^2 \simeq -c_n^{(1,2)} \eta_n \frac{\Lambda^2}{16\pi^2} \simeq -c_n^{(1,2)} \eta_n f^2, \quad (7)$$

where $c_n^{(1,2)}$ are coefficients of order unity and η_i are Yukawa couplings in the right-handed neutrino sector of order $\lesssim 10^{-2}$. Their presence induces negative $\mu_{1,2}^2$ thus triggering the spontaneous breaking of the residual $SU(2) \times U(1)$ gauge flavor symmetry. On the other hand, the size of the induced quartic flavon couplings remains small enough not to significantly affect their mass spectrum [3].

A residual $U(1)$ global symmetry, acting with opposite charge on ϕ_i and $\tilde{\phi}_i$ fields, forbids the generation of mixed terms proportional to $\tilde{\phi}_1^\dagger \phi_2$. Therefore the vacuum which completely breaks the $SU(2) \times U(1)$ gauge flavor symmetry can be parametrized as

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad (8)$$

where v_1 and v_2 are real.

Albeit preserved by the vacuum, the residual global $U(1)_P$ symmetry, is explicitly broken by the lepton Yukawa sector [3] in a way that maintains to a high accuracy the vacuum structure in eq. (8). The scalar potential of the model contains in addition to eq. (5) the standard Higgs potential and flavon-Higgs mixing terms, which, as discussed in ref. [3], neither destabilize the standard electroweak vacuum nor the flavon vacuum considered above.

Let us stress that the only large couplings present in the model—with the exception of the top Yukawa coupling on which we comment in sect. II.B—are the flavor gauge couplings which are taken to be $O(1)$ to make the flavor gauge bosons sufficiently heavy after spontaneous breaking of the symmetry. As we have seen, the scalar sector of the theory is protected from large loop corrections induced by these gauge interactions by a little-Higgs-like mechanism. This feature, together with the presence of small effective Yukawa and scalar couplings (due to vacuum induced suppression factors), makes the model highly stable against radiative corrections from all sectors.

All couplings in the potential and overall fermion scales take natural values (not smaller than 10^{-2}), the only exception being the overall scale of the neutrino mass matrix which implies a Yukawa coupling of $O(10^{-4})$ because of the smallness of the see-saw scale in the model. The large mass hierarchies, present in the quark and lepton sectors only come from the texture generated by the spontaneous breaking of the flavor symmetry.

II. TEXTURES GENERATION

As discussed in ref. [3], the spontaneous breaking of the global $SU(6) \rightarrow Sp(6)$ (approximate) symmetries leads to the breaking of the gauged $[SU(2) \times U(1)]^2$ subgroup to $SU(2) \times U(1)$. Fermions of different families transform according to the $SU(2)_F \times U(1)_F$ gauge flavor symmetry, labeled by the index F in order to distinguish it from the standard electroweak group. In the following, all Greek indices are related to the flavor group while Latin indices refer to the electroweak group.

Textures in the mass matrices of fermions are generated by coupling the flavon fields to the fermions, after the spontaneous breaking of the flavor symmetry. The model does not explain the overall scales of the fermion masses, that have to be put in by hand; it explains the hierarchy among families that exists after that scale has been fixed.

The effective lagrangians are rather cumbersome because many different couplings are allowed by the flavor symmetry. The little flavon fields enter as components of the pseudo-Goldstone field Σ introduced in eq. (1) of the effective non-linear sigma model. In [3] we have shown how the required textures in the leptonic sector arise naturally from the vacuum structure, the precise fit of masses and mixings being determined by detailed values of unknown Yukawa couplings which differ at most by factors of order one. We complete here the discussion by including also the quark sector and presenting a global fit of quark and lepton masses and mixing angles that arises without large hierarchies or fine tuning of the Yukawa coefficients in sect. III.

A. Generalities

After electroweak symmetry breaking, the effective lagrangian contains the following mass terms for fermions:

$$\mathcal{L}^{(m)} = -\bar{\psi}_R^{(i)} M^{(i)} \psi_L^{(i)} - \frac{1}{2} \chi^T C M^{(n)} \chi + H.c., \quad (9)$$

where $\psi_{L,R}^{(i)}$ are chiral fields, $M^{(i)}$ are 3×3 matrices, $i = u, d, l$, $M^{(n)}$ is a 6×6 symmetric matrix, $\chi = (\nu_L, C\nu_R^*)^T$ and flavor indices are understood. C is the charge conjugation matrix.

The neutrino mass matrix can be written in 3×3 block form as:

$$M^{(n)} = \begin{pmatrix} m_L & m_D^T \\ m_D & m_R \end{pmatrix}. \quad (10)$$

In the present case, $m_L = 0$ and the scale of m_R (whose generation does not involve neither electroweak nor flavor symmetry breaking at leading order) is of order f and therefore much larger than that of m_D . In the spirit of effective field theory, one can approximately block-diagonalize $M^{(n)}$, decouple three heavy states which are predominantly standard model singlets, and write the Majorana mass term for the light states as

$$\mathcal{L}^{(m)} = -\frac{1}{2} \nu_L^T C M^{(\nu)} \nu_L + H.c., \quad (11)$$

where now the (symmetric) Majorana mass matrix for the light fields (with some abuse of notation, we identify the light fields with the left-handed components) is $M^{(\nu)} = -m_D^T m_R^{-1} m_D$.

All matrices are non-diagonal in flavor space. One can diagonalize them with appropriate bi-unitary transformations,

$$\text{diag } M^{(i)} = R^{(i)\dagger} M^{(i)} L^{(i)}, \quad (12)$$

$$\text{diag } M^{(\nu)} = L^{(\nu)T} M^{(\nu)} L^{(\nu)}, \quad (13)$$

where $L^{(i)}$, $R^{(i)}$ ($i = u, d, e$) and $L^{(\nu)}$ are 3×3 matrices in flavor space. With these definitions one finds that the mixing matrices appearing in the charge-current interactions according to the standard notation are given by

$$V_{CKM} = L^{(u)\dagger} L^{(d)}, \quad (14)$$

$$V_{PMNS} = L^{(l)\dagger} L^{(\nu)}, \quad (15)$$

for quarks and leptons, respectively. We use the standard definitions of the mixing matrices, in which one writes the down-type quark (neutrino) flavor eigenstates d' (ν') in terms of the mass eigenstates d (ν)—in the basis in which up-type quarks (charged leptons) are diagonal—as

$$d' = V_{CKM} d, \quad (16)$$

$$\nu' = V_{PMNS} \nu. \quad (17)$$

The standard parameterization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in terms of three mixing angles θ_{12} , θ_{13} and θ_{23} and one phase δ reads:

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (18)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. An analogous expression is valid for the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, neglecting the flavor-diagonal Majorana phases.

Quarks are characterized by small mixing angles. In this respect it is natural to consider them as singlets under non-abelian flavor symmetries. We take all standard model quarks—left-handed electroweak doublet components as well as right-handed electroweak singlets—to be singlets under $SU(2)_F$ while being charged under $U(1)_F$. Textures generated by abelian symmetries have been widely discussed in the literature (see for instance [11] and references therein). Here we embed this ansatz in the little flavon framework paying attention to the issue of the stability of the flavon potential, while avoiding the large hierarchies among the Yukawa couplings which are present in the standard model. A possible charge assignment is summarized in Table I.

TABLE I: Summary of the charges of quarks and flavon fields ($\alpha = 2, 3$) under the horizontal flavor groups $SU(2)_F$ and $U(1)_F$. Q_{iL} stands for the electroweak left-handed doublets. q is an arbitrary charge that is not determined.

	$U(1)_F$	$SU(2)_F$
Q_{1L}	$q + 3$	1
Q_{2L}	$q + 2$	1
Q_{3L}	q	1
u_R	$q - 3$	1
c_R	$q - 1$	1
t_R	q	1
d_R	$q - 4$	1
s_R	$q - 2$	1
b_R	$q - 2$	1
$\Sigma_{\alpha-16} = (-i/f \phi_1 + \dots)_{\alpha-1}$	1/2	2
$\Sigma_{\alpha-13} = (+i/f \phi_2 + \dots)_{\alpha-1}$	-1/2	2
$\Sigma_{32+\alpha} = (-i/f \phi_1^* + \dots)_{\alpha-1}$	-1/2	2*
$\Sigma_{62+\alpha} = (-i/f \phi_2^* + \dots)_{\alpha-1}$	1/2	2*

Given the charges in Table I, we find for the up quarks the following effective Yukawa lagrangian

$$\begin{aligned}
 -\mathcal{L}_u = & \lambda_{31} \bar{t}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^3 \tilde{H}^\dagger Q_{1L} + \lambda_{32} \bar{t}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^2 \tilde{H}^\dagger Q_{2L} \\
 & + \bar{t}_R (\lambda_{33} + \lambda'_{33} \Sigma_{\alpha-16} \Sigma_{32+\alpha} + \lambda''_{33} \Sigma_{\alpha-13} \Sigma_{62+\alpha}) \tilde{H}^\dagger Q_{3L} \\
 & + \lambda_{21} \bar{c}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^4 \tilde{H}^\dagger Q_{1L} + \lambda_{22} \bar{c}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^3 \tilde{H}^\dagger Q_{2L} \\
 & + \lambda_{23} \bar{c}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha}) \tilde{H}^\dagger Q_{3L} \\
 & + \lambda_{11} \bar{u}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^6 \tilde{H}^\dagger Q_{1L} + \lambda_{12} \bar{u}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^5 \tilde{H}^\dagger Q_{2L} \\
 & + \lambda_{13} \bar{u}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^3 \tilde{H}^\dagger Q_{3L} + H.c.
 \end{aligned} \tag{19}$$

as well as

$$\begin{aligned}
 -\mathcal{L}_d = & \tilde{\lambda}_{31} \bar{b}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^5 H^\dagger Q_{1L} + \tilde{\lambda}_{32} \bar{b}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^4 H^\dagger Q_{2L} \\
 & + \tilde{\lambda}_{33} \bar{b}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^2 H^\dagger Q_{3L} \\
 & + \tilde{\lambda}_{21} \bar{s}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^5 H^\dagger Q_{1L} + \tilde{\lambda}_{22} \bar{s}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^4 H^\dagger Q_{2L} \\
 & + \tilde{\lambda}_{23} \bar{s}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^2 H^\dagger Q_{3L} \\
 & + \tilde{\lambda}_{11} \bar{d}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^7 H^\dagger Q_{1L} + \tilde{\lambda}_{12} \bar{d}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^6 H^\dagger Q_{2L} \\
 & + \tilde{\lambda}_{13} \bar{d}_R (\Sigma_{\alpha-13} \Sigma_{32+\alpha})^4 H^\dagger Q_{3L} + H.c.
 \end{aligned} \tag{20}$$

for the down quarks.

Notice that even though the t quark has a large Yukawa coupling that could introduce a potentially destabilizing term in the flavon effective potential, the contribution to the flavon mass terms of t -quark loops induced by the couplings in eq. (19)

$$\mu_{1,2}^2 \simeq -\text{Re} (\lambda_{33}^* (\lambda'_{33}, \lambda''_{33})) \frac{\langle h_0 \rangle^2}{f^2} \frac{\Lambda^2}{16\pi^2} \tag{21}$$

remains negligible compared to eq. (7).

C. Leptons

At variance with the quark sector, lepton flavor mixings differ by the presence of large angles, as implied by the neutrino oscillation data. The maximal atmospheric neutrino oscillation together with the hierarchy structure in the charged lepton mass spectrum, suggest that the leptons of the second and third family may belong to flavor doublets. In the following we take the weak electron doublet l_{eL} to be an $SU(2)_F$ singlet charged under $U(1)_F$, while the standard model doublets $l_{\mu,\tau L}$ are members of a doublet in flavor space. Right-handed charged leptons are assumed to follow a similar structure.

In order to have a *see-saw*-like mechanism [12], we introduce three right-handed neutrinos ν_R^i which are $SU(2)_F$ singlets. This choice allows us to take right-handed neutrino mass entries at the scale $M \sim f$. Table II summarizes the charge assignments.

TABLE II: Summary of the charges of all leptons under the horizontal flavor groups $SU(2)_F$ and $U(1)_F$.

	$U(1)_F$	$SU(2)_F$
l_{eL}	-2	1
e_R	2	1
$L_L = (l_\mu, l_\tau)_L$	1/2	2
$E_R = (\mu, \tau)_R$	1/2	2
ν_{1R}	1	1
ν_{2R}	-1	1
ν_{3R}	0	1

The neutrino lagrangian is obtained after integrating out the three right-handed neutrinos and, at the leading order in the right-handed neutrino mass and in number of Σ fields, is given by (see [3]):

$$\begin{aligned}
-2\mathcal{L}_\nu &= \frac{(\overline{l_{1L}^c} \tilde{H}^*)(\tilde{H}^\dagger l_{1L})}{M} [2\lambda_{1\nu}\lambda_{2\nu} + r \lambda_{3\nu}^2] [\Sigma_{\alpha-16}\Sigma_{62+\alpha}]^{-2Y_{1L}} \\
&+ \frac{(\overline{l_{1L}^c} \tilde{H}^*)(\tilde{H}^\dagger l_{\alpha L}) + (\overline{l_{\alpha L}^c} \tilde{H}^*)(\tilde{H}^\dagger l_{1L})}{M} \lambda_{2\nu}(\lambda'_{1\nu} \epsilon_{\alpha\beta}\Sigma_{\beta-16} + \lambda''_{1\nu} \Sigma_{62+\alpha}) [\Sigma_{\delta-16}\Sigma_{62+\delta}]^{-Y_{1L}+Y_{\nu_{2R}}} \\
&+ \frac{(\overline{l_{\alpha L}^c} \tilde{H}^*)(\tilde{H}^\dagger l_{\beta L})}{2M_3} (i\sigma_2\sigma_\tau)_{\alpha\beta} (i\sigma_2\sigma_\tau)_{\delta\gamma} [(\lambda'_{3\nu})^2 \Sigma_{\delta-13}\Sigma_{\gamma-13} \\
&+ \lambda'_{3\nu}\lambda''_{3\nu} (\epsilon_{\gamma\gamma'}\Sigma_{\delta-13}\Sigma_{32+\gamma'} + \delta \leftrightarrow \gamma) + (\lambda''_{3\nu})^2 \epsilon_{\delta\delta'}\epsilon_{\gamma\gamma'}\Sigma_{32+\delta'}\Sigma_{32+\gamma'}] + H.c., \tag{22}
\end{aligned}$$

where $r = M/M_3$, M and M_3 are the masses of the right-handed neutrinos, $\sigma_\tau/2$ are the generators of the $SU(2)_f$ gauge group ($\tau = 1, 2, 3$).

The lagrangian for the charged leptons is given by

$$\begin{aligned}
\mathcal{L}_e &= \overline{e_R} \left[\lambda_{1e} (\Sigma_{\alpha-16}\Sigma_{62+\alpha})^{(-Y_{1L}+Y_{1R})} (H^\dagger l_{1L}) \right. \\
&\quad \left. + i (\lambda_{3e} \Sigma_{62+\alpha} + \lambda_{2e} \epsilon_{\alpha\beta}\Sigma_{\beta-16})(\Sigma_{\delta-16}\Sigma_{62+\delta})^{(Y_{1R}-1)} (H^\dagger l_{\alpha L}) \right] \\
&+ \overline{E_{\alpha R}} \left[i (\lambda'_{1E} \Sigma_{62+\alpha} + \lambda_{1E} \epsilon_{\alpha\beta}\Sigma_{\beta-16})(\Sigma_{\delta-16}\Sigma_{62+\delta})^{-Y_{1L}} (H^\dagger l_{1L}) \right. \\
&\quad \left. + \overline{E_{\alpha R}} \left[\delta_{\alpha\beta} (-\lambda_{2E} + \lambda'_{2E} \Sigma_{\gamma-16}\Sigma_{32+\gamma} + \lambda''_{2E} \Sigma_{\gamma-13}\Sigma_{62+\gamma}) \right. \right. \\
&\quad \left. \left. + (\lambda_{3E} \Sigma_{\alpha-16}\Sigma_{32+\beta} + \lambda'_{3E} \epsilon_{\alpha\delta}\epsilon_{\beta\gamma} \Sigma_{62+\delta}\Sigma_{\gamma-13} + (3 \leftrightarrow 6)) \right. \right. \\
&\quad \left. \left. + (\lambda_{4E} \Sigma_{\alpha-16}\Sigma_{\gamma-13}\epsilon_{\beta\gamma} + \lambda'_{4E} \epsilon_{\alpha\delta}\Sigma_{6,2+\delta}\Sigma_{3,2+\alpha} + (3 \leftrightarrow 6)) \right] (H^\dagger l_{\beta L}) + H.c. \right] \tag{23}
\end{aligned}$$

On the vacuum that completely breaks the the $SU(2)_F \times U(1)_F$ gauge symmetry, the little flavons acquire expectation values $v_{1,2} = \varepsilon_{1,2} f$, with $\varepsilon_{1,2} < 1$. By inspection of the Yukawa lagrangians introduced in the previous section, we can determine the fermion mass matrices.

Since all quarks are $SU(2)_F$ singlets the all entries of their mass matrices are proportional to powers of $k \equiv \varepsilon_1 \varepsilon_2$. Due to the large number of possible higher-order terms, we only take for each entries the first non-vanishing term, and obtain

$$M^{(u)} = \langle h_0 \rangle \begin{pmatrix} \lambda_{11} k^6 & \lambda_{12} k^5 & \lambda_{13} k^3 \\ \lambda_{21} k^4 & \lambda_{22} k^3 & \lambda_{23} k \\ \lambda_{31} k^3 & \lambda_{32} k^2 & \lambda_{33} \end{pmatrix} \quad (24)$$

and

$$M^{(d)} = \langle h_0 \rangle k^2 \begin{pmatrix} \tilde{\lambda}_{11} k^5 & \tilde{\lambda}_{12} k^4 & \tilde{\lambda}_{13} k^2 \\ \tilde{\lambda}_{21} k^3 & \tilde{\lambda}_{22} k^2 & \tilde{\lambda}_{23} \\ \tilde{\lambda}_{31} k^3 & \tilde{\lambda}_{32} k^2 & \tilde{\lambda}_{33} \end{pmatrix}. \quad (25)$$

The essential feature of the previous mass matrices is that the fundamental textures are determined by the vacuum structure alone—that is that obtained by taking all Yukawa couplings λ_{ij} and $\tilde{\lambda}_{ij}$ of $O(1)$. In fact, by computing the corresponding CKM matrix one finds in first approximation

$$V_{\text{CKM}} = \begin{pmatrix} 1 & O(k) & O(k^3) \\ O(k) & 1 & O(k^2) \\ O(k^3) & O(k^2) & 1 \end{pmatrix}, \quad (26)$$

that is roughly of the correct form and, moreover, suggests a value of $k \simeq \sin \theta_C \simeq 0.2$.

At the same time it is possible to extract from (24) and (25) approximated mass ratios:

$$\frac{m_u}{m_c} \simeq \frac{m_c}{m_t} \simeq \frac{m_d}{m_s} \simeq O(k^3) \quad \frac{m_s}{m_b} \simeq O(k^2) \quad (27)$$

which again roughly agree with the experimental values.

These results show that the quark masses and mixing angles can be reproduced by our textures. While a rough agreement is already obtained by taking all Yukawa coupling to be equal, the precise agreement with the experimental data depends on the actual choice of the Yukawa couplings λ_{ij} and $\tilde{\lambda}_{ij}$. However, their values can be taken all of the same order, as we shall see in last section.

Notice that the textures used in this work do not satisfactorily address the flavor problem in a supersymmetric framework: the abelian nature of the flavor symmetry in the quark sector, and the large mixing angles in the right handed mixing matrices $R^{(i)}$ would in general induce large contributions to FCNC processes via diagram with gluino exchange. The diagonal entries of the squark mass matrices are not forbidden by the abelian symmetry, and in general one expects all of them to be determined only by the scale of supersymmetry breaking, up to $\mathcal{O}(1)$ coefficients. Once fermions are diagonalized, large off-diagonal entries are generated in the 3×3 right-handed down-type squark mass matrix, because of the large mixing angles in $R^{(d)}$ (this can be easily seen from the fact that second and third row of eq. (25) have entries of the same order). Phenomenologically, for generic choices of the diagonal elements of the squark mass matrices, this leads to contributions to $\Delta F = 2$ processes (K^0 - \bar{K}^0 or B^0 - \bar{B}^0 mixings and related CP violating observables) largely in excess of the experimental data [9]. This can be avoided allowing for a degeneracy of the diagonal entries themselves, albeit with a tuning at least at the percent level.

In the lepton sector, the VEV's of the little flavons gives us the left-handed neutrino and charged-lepton mass matrices (again, we only retain the first non-vanishing term for each entry):

$$M^{(\nu)} = \frac{\langle h_0 \rangle^2}{M} \begin{pmatrix} [r \lambda_{3\nu}^2 + 2\lambda_{1\nu}\lambda_{2\nu}] \varepsilon_1^4 \varepsilon_2^4 & -\lambda_{2\nu} \lambda'_{1\nu} \varepsilon_1^2 \varepsilon_2 & -\lambda_{2\nu} \lambda''_{1\nu} \varepsilon_1 \varepsilon_2^2 \\ -\lambda_{2\nu} \lambda'_{1\nu} \varepsilon_1^2 \varepsilon_2 & r \lambda_{3\nu}^2 \varepsilon_2^2 & r \lambda'_{3\nu} \lambda''_{3\nu} \varepsilon_1 \varepsilon_2 \\ -\lambda_{2\nu} \lambda''_{1\nu} \varepsilon_1 \varepsilon_2^2 & r \lambda'_{3\nu} \lambda''_{3\nu} \varepsilon_1 \varepsilon_2 & r \lambda_{3\nu}^2 \varepsilon_1^2 \end{pmatrix}. \quad (28)$$

The eigenvalues of this matrix are the masses of the three neutrinos. The scale M is just below or around f and therefore we are not implementing the usual see-saw mechanism that requires scales as large as 10^{13} TeV. Therefore, realistic neutrino masses are obtained by tuning the corresponding effective Yukawa couplings to the order of 10^{-4} (which are the smallest couplings in the model).

In the same approximation, the Dirac mass matrix for the charged leptons is given by

$$M^{(l)} = \langle h_0 \rangle \begin{pmatrix} \lambda_{1e} \varepsilon_1^4 \varepsilon_2^4 & \lambda_{2e} \varepsilon_1^2 \varepsilon_2 & \lambda_{3e} \varepsilon_1 \varepsilon_2^2 \\ \lambda_{1E} \varepsilon_1^2 \varepsilon_2^3 & \lambda_{2E} & (\lambda'_{14E} + \lambda'_{24E}) \varepsilon_1 \varepsilon_2 \\ \lambda'_{1E} \varepsilon_1^3 \varepsilon_2^2 & -(\lambda_{14E} + \lambda_{24E}) \varepsilon_1 \varepsilon_2 & \lambda_{2E} \end{pmatrix}, \quad (29)$$

where the notation follows that of eq. (39) in ref. [3].

In order to exhibit the main features of the underlying textures, we study the limit

$$\varepsilon_1 \rightarrow 1 \quad \text{and} \quad \varepsilon_2 \rightarrow k, \quad (30)$$

which is suggested by the additional constraint $\varepsilon_1 \varepsilon_2 \simeq \sin \theta_C$, obtained from the study of the quark textures.

Notice that in ref. [3] we have considered a slightly different charged-lepton texture that accounts for maximal mixing in the limit $\varepsilon_1^2 \ll \varepsilon_2^2 \ll 1$ (or, equivalently $\varepsilon_2^2 \ll \varepsilon_1^2 \ll 1$).

In the limit (30), the matrices in eqs. (28)–(29) reduce—at the order $O(k^2)$, and up to overall factors—to

$$M^{(\nu)} = \begin{pmatrix} 0 & O(k) & O(k^2) \\ O(k) & O(k^2) & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix} \quad \text{and} \quad M^{(l)} = \begin{pmatrix} 0 & O(k) & O(k^2) \\ 0 & 1 & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix}, \quad (31)$$

where, as before, the 1 stands for $O(1)$ coefficients.

The eigenvalues of $M^{(l)}$ can be computed by diagonalizing $M^{(l)\dagger} M^{(l)}$. This product is—again for each entry to leading order in k :

$$M^{(l)\dagger} M^{(l)} = \begin{pmatrix} 0 & 0 & O(k^2) \\ 0 & 1 & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix}. \quad (32)$$

By inspection of the 2×2 sub-blocks, the matrix eq. (32) is diagonalized by three rotations with angles, respectively, $\theta_{23}^l \simeq \pi/4$ and $\theta_{12}^l \simeq \theta_{13}^l \ll 1$, leading to one maximal mixing angle and two minimal. On the other hand, the neutrino mass matrix in eq. (31) is diagonalized by three rotations with angles, respectively, $\tan 2\theta_{12}^\nu \simeq 2/k$ and $\theta_{23}^\nu \simeq \theta_{13}^\nu \ll 1$ (the label 3 denotes the heaviest eigenstate). Therefore, the textures in the mass matrices in eqs. (28)–(29) give rise to a PMNS mixing matrix—that is the combination of the two rotations above—in which θ_{23} is maximal, θ_{12} is large (up to maximal), while θ_{13} remains small.

The natural prediction when taking all coefficients $O(1)$ is then: a large atmospheric mixing angle θ_{23} , possibly maximal, another large solar mixing angle θ_{12} , and a small θ_{13} mixing angle; at the same time, the mass spectrum includes one light ($O(k^4)$) and two heavy states ($O(1)$) in the charged lepton sector (m_e , m_μ and m_τ respectively), two light states ($O(k^2)$) and one heavy ($O(1)$) in the neutrino sector, thus predicting a neutrino spectrum with normal hierarchy.

By flavor symmetry, one expects masses of the same order of magnitude for μ and τ . The ratio of the masses of τ and μ is given to $O(k)$ by:

$$R \equiv \frac{m_\mu}{m_\tau} \simeq \frac{\sqrt{\det(m^{(l)\dagger} m^{(l)})}}{\text{Tr}(m^{(l)\dagger} m^{(l)})}, \quad (33)$$

where $m^{(l)}$ is the μ - τ sub-matrix of $M^{(l)}$. The experimental splitting can be explained only admitting a moderate amount of fine-tuning, of a factor 10, between the coefficients of the charge lepton mass matrix such as to make $R \simeq O(10^{-1})$. One can quantify the stability of this fine-tuning with the logarithmic derivatives $d_{Y_{ij}}^R$ of this ratio with respect to the corresponding Yukawa coefficients Y_{ij} [21]:

$$d_{Y_{ij}}^R \equiv \left| \frac{Y_{ij}}{R} \frac{\partial R}{\partial Y_{ij}} \right|. \quad (34)$$

Using the experimental value $R = m_\mu/m_\tau$, and the numerical solution given in sect. III.A, we find $(i, j = 2, 3)$ $d_{Y_{ij}}^R < 5$, where the largest value arises because of the leading order correlation between the diagonal Yukawa entries ($Y_{22} = Y_{33}$) in the charge lepton sector that doubles the sensitivity. In the absence of any fine-tuning one would expect values of $d_{Y_{ij}}^R$ at most around unity. Nevertheless, the tree level value of R is not destabilized by Yukawa radiative corrections, since they are very suppressed in the model.

The ultraviolet completion of the theory, in which all effective couplings should be computed from a restrict number of fundamental parameters, might explain possible correlations among the Yukawa couplings, together with the suppression of the the overall neutrino scale.

Finally, notice that even though the quark charges leave an undetermined factor q (see Table I), gauge anomalies are present in the theory, as it can be easily seen by inspection considering the charges of the matter fields. They can be cancelled by adding appropriate Wess-Zumino terms [10].

III. FITTING THE DATA

Let us first briefly review the experimental data and comment on the possible range of values we consider acceptable in reproducing these data within the model.

The CKM matrix is rather well known as are the masses of the quarks (see, e.g., the PDG [13]). We will estimate only ratios of masses which are renormalization group invariant, so that we only have to be careful in computing them at a common scale. Taking into account the uncertainties in the values of the quark masses, the mass ratio we would like the model to reproduce are given by

$$\frac{m_t}{m_c} = 248 \pm 70 \quad \frac{m_b}{m_s} = 40 \pm 10 \quad \frac{m_s}{m_d} = 430 \pm 300 \quad \frac{m_c}{m_u} = 325 \pm 200. \quad (35)$$

The CKM phase is determined [14] to be

$$\delta = 61.5^\circ \pm 7^\circ \quad (\sin 2\beta = 0.705_{-0.032}^{+0.042}). \quad (36)$$

Compelling evidences in favor of neutrino oscillations and, accordingly of non-vanishing neutrino masses has been collected in recent years from neutrino experiments [15]. Combined analysis of the experimental data show that the neutrino mass matrix is characterized by a hierarchy with two square mass differences (at 99.73% CL):

$$\begin{aligned} \Delta m_\odot^2 &= (5.3 - 17) \times 10^{-5} \text{eV}^2 \\ |\Delta m_\oplus^2| &= (1.4 - 3.7) \times 10^{-3} \text{eV}^2, \end{aligned} \quad (37)$$

the former controlling solar neutrino oscillations [16] and the latter the atmospheric neutrino experiments [17]. In the context of three active neutrino oscillations, the mixing is described by the PMNS mixing matrix V_{PMNS} in eq. (15). Such a matrix is parameterized by three mixing angles, two of which (θ_{12} and θ_{23}) can be identified with the mixing angles determining solar [16] and atmospheric [17] oscillations, respectively (again, at 99.73% CL):

$$\begin{aligned} \tan^2 \theta_\odot &= 0.23 - 0.69, \\ \sin^2 2\theta_\oplus &= 0.8 - 1.0. \end{aligned} \quad (38)$$

For the third angle, controlling the mixing $\nu_\tau - \nu_e$, there are at present only upper limits, deduced by reactor neutrino experiments [18] (at 99.73% CL):

$$\sin^2 \theta_{13} < 0.09. \quad (39)$$

Other observable quantities determined by the neutrino mass matrix have not been measured yet. These include: 1) the type of neutrino spectrum, with normal or inverted hierarchy (see for instance [19] for a definition), 2) the common mass scale, i.e. the actual value of the lowest mass eigenvalue m_1 , 3) the (Dirac) phase δ^l responsible for CP violation in leptonic flavor changing processes, 4) the two Majorana flavor-diagonal CP-violating phases, 5) the sign of $\cos 2\theta_\oplus$. Several proposal appeared in the literature to measure all these quantities in the next generation neutrino experiments, together with the mixing angle θ [20]. Our model predicts a neutrino spectrum with normal hierarchy, with a very small mass for the lighter neutrinos $m_{1,2} \ll \sqrt{\Delta m_\oplus^2}$.

Finally, the values of the charged-lepton masses are given by $m_\tau \simeq 1777$ MeV, $m_\mu \simeq 106$ MeV and $m_e \simeq 0.51$ MeV, respectively. We therefore have

$$\frac{m_\tau}{m_\mu} \simeq 17, \quad \frac{m_\mu}{m_e} \simeq 207, \quad \frac{m_\tau}{m_e} \simeq 3484. \quad (40)$$

In order to show that the model reproduces in a natural manner all the experimental data we retain the first non-vanishing contribution to each entry in all mass matrices and then—having extracted an overall coefficient for each matrix according to eqs. (24)–(25) and eqs. (28)–(29)—treat the ratios of Yukawa couplings as a set of arbitrary parameters to be varied within a $O(1)$ range.

We keep the VEV's v_1 and v_2 fixed at the values obtained by taking $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.2$.

In practice, we generated for the quark matrices many sets of 18 complex Yukawa parameters whose moduli differ by at most a factor 10 and accepted those that reproduces the known masses and mixings. As an example, we found that the assignments

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} = \lambda_U \begin{bmatrix} 1.2 + 0.073i & 1.9 + 0.31i & -0.82 + 1.3i \\ -0.32 - 0.41i & -0.58 + 0.85i & -0.48 - 0.95i \\ 1.2 + 0.84i & -1.5 + 0.78i & 1.4 + 0.72i \end{bmatrix} \quad (41)$$

and

$$\begin{bmatrix} \tilde{\lambda}_{11} & \tilde{\lambda}_{12} & \tilde{\lambda}_{13} \\ \tilde{\lambda}_{21} & \tilde{\lambda}_{22} & \tilde{\lambda}_{23} \\ \tilde{\lambda}_{31} & \tilde{\lambda}_{32} & \tilde{\lambda}_{33} \end{bmatrix} = \lambda_D \begin{bmatrix} -0.55 - 1.5i & -0.76 - 0.42i & 0.55 + 1.2i \\ -1.3 - 0.83i & 0.32 + 1.2i & 0.58 + 0.67i \\ 0.75 - 1.0i & -1.4 + 0.17i & 0.09 - 1.6i \end{bmatrix} \quad (42)$$

with λ_U and λ_D of $O(1)$, give masses and mixing angles in excellent agreement with the experimental data. We have followed a similar procedure for the leptonic sector, generating random sets of 13 real parameters. Lacking experimental signature of CP violation in the leptonic sector, we have neglected, for the purpose of illustration, leptonic phases in the numerical exercise. Again, we obtain that for the representative choice

$$\begin{bmatrix} r \lambda_{3\nu}^2 + 2\lambda_{1\nu}\lambda_{2\nu} & -\lambda_{2\nu}\lambda'_{1\nu} & -\lambda_{2\nu}\lambda''_{1\nu} \\ -\lambda_{2\nu}\lambda'_{1\nu} & r \lambda_{3\nu}^2 & r \lambda'_{3\nu}\lambda''_{3\nu} \\ -\lambda_{2\nu}\lambda'_{1\nu} & r \lambda'_{3\nu}\lambda''_{3\nu} & r \lambda_{3\nu}^2 \end{bmatrix} = \lambda_\nu^2 \begin{bmatrix} 0.66 & -1.0 & 2.9 \\ -1.0 & 1.9 & 0.29 \\ 2.9 & 0.29 & -1.1 \end{bmatrix} \quad (43)$$

with $\lambda_\nu = O(10^{-4})$, and

$$\begin{bmatrix} \lambda_{1e} & \lambda_{2e} & \lambda_{3e} \\ \lambda_{1E} & \lambda_{2E} & \lambda'_{4E} \\ \lambda'_{1E} & -\lambda_{4E} & \lambda_{2E} \end{bmatrix} = \lambda_E \begin{bmatrix} 1.2 & 0.27 & 1.4 \\ -1.2 & 0.39 & 2.3 \\ 0.36 & 2.0 & 0.39 \end{bmatrix} \quad (44)$$

with $\lambda_E = O(10^{-2})$, the experimental values are well reproduced.

Table III summarizes the experimental data and compares them to the result of the above procedure. The agreement is quite impressive, keeping in mind that we have varied only the leading terms in the mass matrices. While the values of the overall constants (which are related to the scale of the heaviest state in the mass matrices) are not explained by the model, the hierarchy among the mass eigenvalues and the mixing angles are given in first approximation by the flavor symmetry and the flavor vacuum so that, within each sector, the Yukawa couplings remain in a natural range.

The phenomenology related to the gauge boson and flavon couplings with quark and leptons will be studied in a forthcoming work where direct bounds for the masses of the flavons will be derived. We expect these to come mainly from flavor gauge mediated processes since the flavon effective couplings to matter are suppressed by powers of weak and flavor vacuum expectations values over the flavon scale f .

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TABLE III: Experimental data vs. the result of our numerical analysis based on a representative set of Yukawa couplings of order one (see text) and $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.2$. Uncertainties in the experimental inputs are explained in the main body.

	exp	numerical results
$ V_{us} $	$0.219 - 0.226$	0.22
$ V_{ub} $	$0.002 - 0.005$	0.0035
$ V_{cb} $	$0.037 - 0.043$	0.040
$ V_{td} $	$0.004 - 0.014$	0.0079
$ V_{ts} $	$0.035 - 0.043$	0.039
δ	$61.5^\circ \pm 7^\circ$	61°
$\sin 2\beta$	$0.705^{+0.042}_{-0.032}$	0.69
m_t/m_c	248 ± 70	219
m_c/m_u	325 ± 200	300
m_b/m_s	40 ± 10	45
m_s/m_d	430 ± 300	231
$\tan^2 \theta_\odot$	$0.23 - 0.69$	0.32
$\sin^2 2\theta_\oplus$	$0.8 - 1.0$	1.0
$\sin^2 \theta_{13}$	< 0.09	0.08
$\Delta m_\odot^2 / \Delta m_\oplus^2$	$0.014 - 0.12$	0.043
m_τ/m_μ	17	15
m_μ/m_e	207	231
m_τ/m_e	3484	3465

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